

Sea quark effects in B_K from $N_f = 2$ clover-improved Wilson fermions

UKQCD collaboration

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Background

- $\langle \overline{K^0} | Q^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$
- $\varepsilon_K = \frac{\mathcal{A}(K_L \rightarrow (\pi\pi)_{I=0})}{\mathcal{A}(K_S \rightarrow (\pi\pi)_{I=0})} = \text{const.} \times \hat{B}_K \times \bar{\eta}(1 - \bar{\rho})$
- B_K is the dominant uncertainty
- LAT03 plenary concluding remarks (D Becirevic): Sharpe's estimate of 15% quenching error remains ...

$$B_K(\overline{\text{MS}}, 2 \text{ GeV}) = 0.63(4)(\pm 15\%)$$

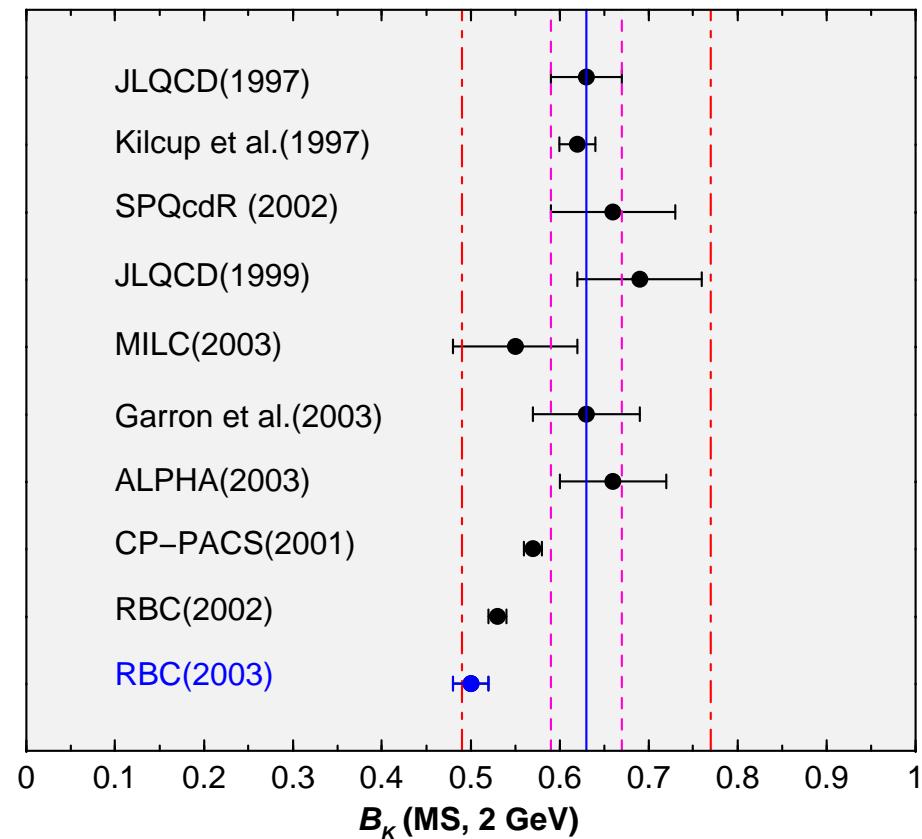
$$\hat{B}_K = 0.87(6)(13)$$

- Addressing this would justify all the hard work in bringing down the other errors

Previous calculations

	$B_K(\overline{\text{MS}}, 2 \text{ GeV})$	Action	Ren	a^{-1}/GeV
JLQCD (97)	0.63(4)	Stag	PT	∞
Kilcup <i>et al.</i> (97)	0.62(2)(2)	Stag	PT	∞
SPQcdR (02)	0.66(7)	Clover	NP	∞
JLQCD (99)	0.69(7)	Wilson	NP	∞
MILC (03)	0.55(7)	O'lap	PT	∞
Garron <i>et al.</i> (03)	0.63(6)(1)	O'lap	NP	2.1
ALPHA (03)	0.66(6)(2)	TM	NP	2.1
CPPACS (01)	0.57(1)	DW	PT	1.8,2.8
RBC (02)	0.53(1)	DW	NP	1.9
RBC (03)	0.50(2)	dyn DW	NP	1.8

Previous calculations



- Quenched value settling down around JLQCD (1997) staggered value of 0.63(4)
- Only complete unquenched number from RBC with DW is smallest, even cf already lower quenched DW numbers
- Other unquenched numbers, on limited parameters, consistent with quenched

This work

Set	β	c_{SW}	κ_{sea}	$\frac{a}{\text{fm}}$	$\frac{a^{-1}}{\text{GeV}}$	$\frac{m_P}{m_V}$ uni	cfgs
I	5.20	2.0171	0.1350	0.103	1.91	0.70	100
II	5.26	1.9497	0.1345	0.104	1.90	0.78	100
III	5.29	1.9192	0.1340	0.102	1.94	0.83	80

- UKQCD configurations small volume 32×16^3
- Non-perturbatively $\mathcal{O}(a)$ -improved Wilson fermions
- Lattice spacings matched using fixed r_0/a (independent of valence quark)
- Five valence quarks for each set
- All work done using FermiQCD

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- Proximity of a phase transition with large cutoff effects?
- Variation of spacing with sea quark mass not well understood
- Nevertheless, three sets of configurations with *matched* lattice spacing and varying sea quark mass
- Degenerate quarks: no $SU(3)_f$ -breaking, but expect small effect

Method I

Following Crisafulli *et al.* [hep-lat/9509029], SPQcdR [hep-lat/0012009]

$$\langle \bar{P}^0, \vec{p} | Q(\mu) | P^0, \vec{q} \rangle = \alpha' + \beta' m_P^2 + \delta' m_P^4 + \\ (p \cdot q) [\gamma + \gamma' + (\epsilon + \epsilon') m_P^2 + (\xi + \xi')(p \cdot q)] + \dots$$

$$R_3(\vec{p}) = \frac{\mathcal{C}^{(3)}(t_x, t_y; p_x, p_y; \mu)}{Z_A^2 \mathcal{C}_{PP}^{(2)}(t_x; p_x) \mathcal{C}_{PP}^{(2)}(t_y; p_y)} \longrightarrow \frac{1}{Z_A^2 \mathcal{Z}_P^2} \langle \bar{P}^0, \vec{p}_x | Q(\mu) | P^0, \vec{p}_y \rangle,$$

$$X(0) = \frac{8}{3} \left| \frac{\mathcal{C}_{A_0 P}^{(2)}(t_x)}{\mathcal{C}_{PP}^{(2)}(t_x)} \right|^2 \longrightarrow \frac{1}{Z_A^2 \mathcal{Z}_P^2} \frac{8}{3} f_P^2 \textcolor{blue}{m_P^2},$$

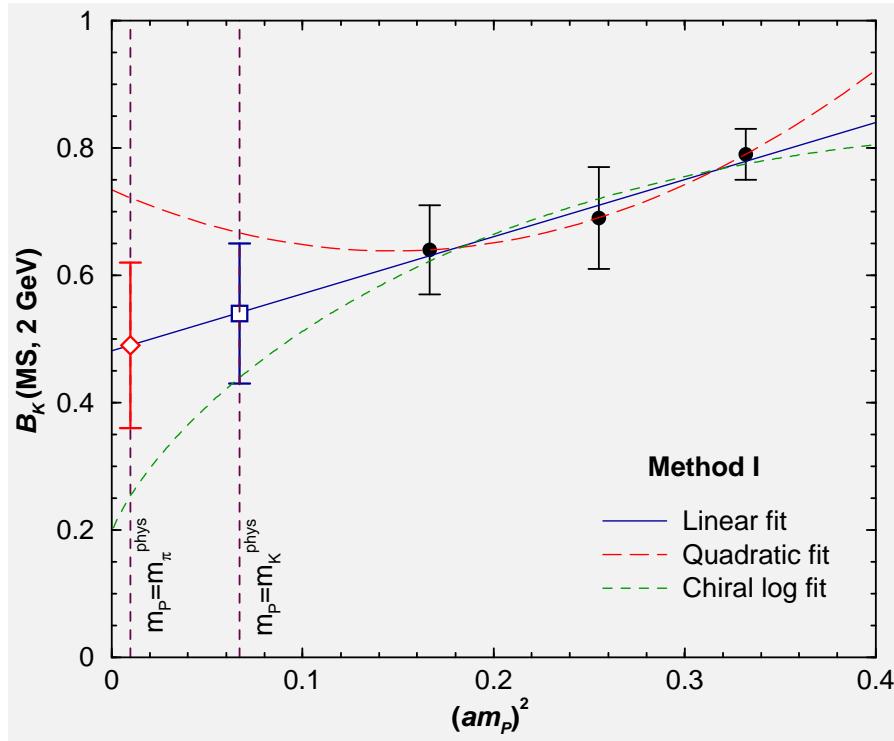
$$X(\vec{p}) = X(0) \cdot \frac{(p_x \cdot p_y)}{m_P^2} \longrightarrow \frac{1}{Z_A^2 \mathcal{Z}_P^2} \frac{8}{3} f_P^2 (\textcolor{blue}{p}_x \cdot \textcolor{blue}{p}_y)$$

$$R_3(\vec{p}) = \tilde{\alpha}' + \tilde{\beta}' \textcolor{blue}{X}(0) + (\tilde{\gamma} + \tilde{\gamma}') \textcolor{blue}{X}(\vec{p})$$

with

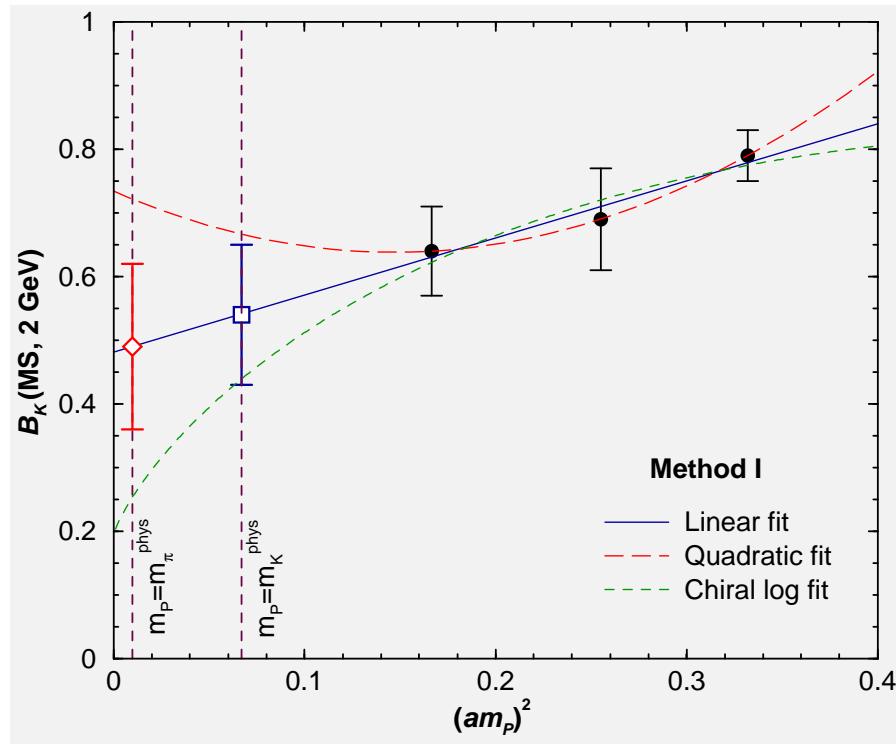
$$\tilde{\alpha}' \equiv \frac{\alpha'}{Z_A^2 \mathcal{Z}_P^2}, \quad \tilde{\beta}' \equiv \frac{3\beta'}{8f_P^2}, \quad \tilde{\gamma} \equiv \frac{3\gamma}{8f_P^2} \quad \text{and} \quad \tilde{\gamma}' \equiv \frac{3\gamma'}{8f_P^2},$$

Method I



- One number for each sea quark, valence quarks in the simulated region
- These values are in the region of the quenched numbers

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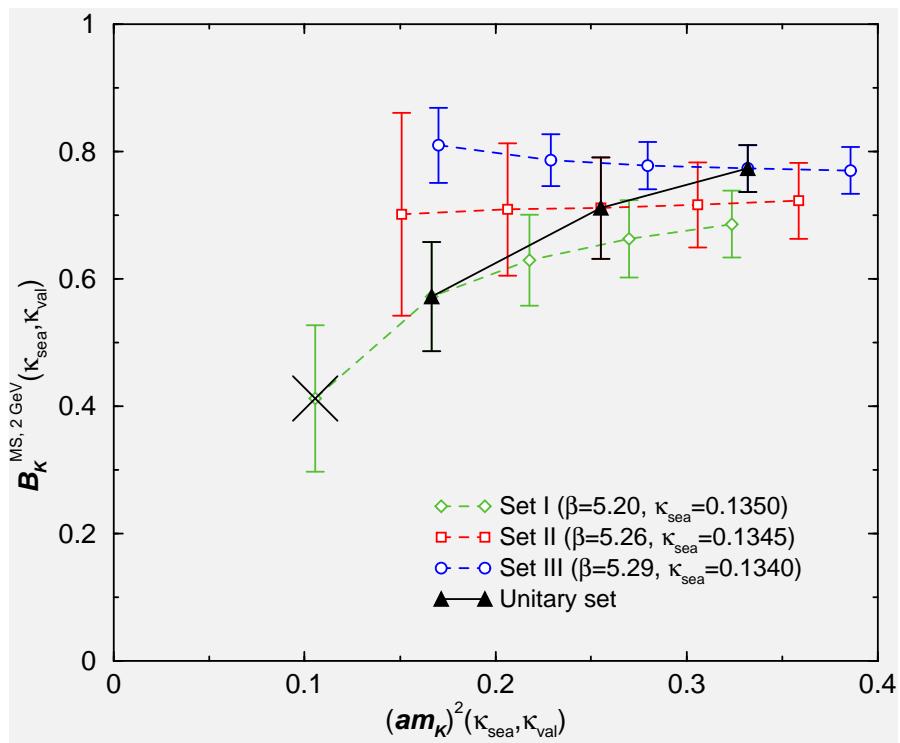
$$B_K(\overline{\text{MS}}, 2 \text{ GeV}) = 0.49(13)$$

- One number for each sea quark, valence quarks in the simulated region
- These values are in the region of the quenched numbers
- Extrapolate $m_{\text{sea}} \rightarrow m_{u,d}$ to get value for **realistic sea** quarks but **valence** quarks in simulated region, $\approx 2m_s$

Method II

$$\left. \frac{R_3(\vec{p}) - R_3(0)}{X(\vec{p}) - X(0)} \right|_{\kappa_{\text{sea}}, \kappa_{\text{val}}} = B_K(\kappa_{\text{sea}}, \kappa_{\text{val}})$$

Following Gupta *et al.*,
hep-lat/9210018,
hep-lat/9611023

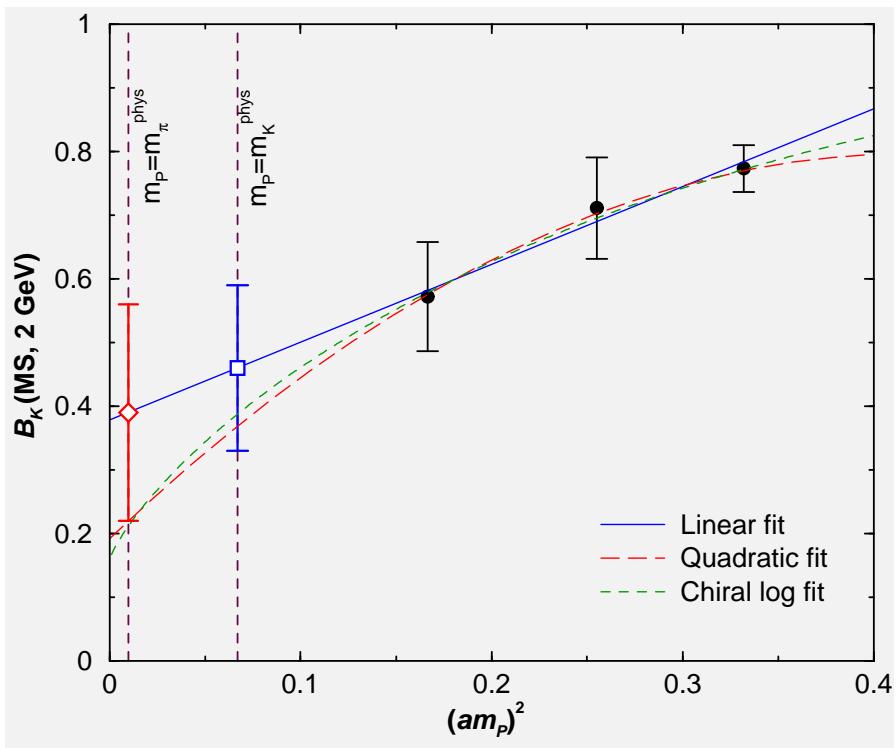


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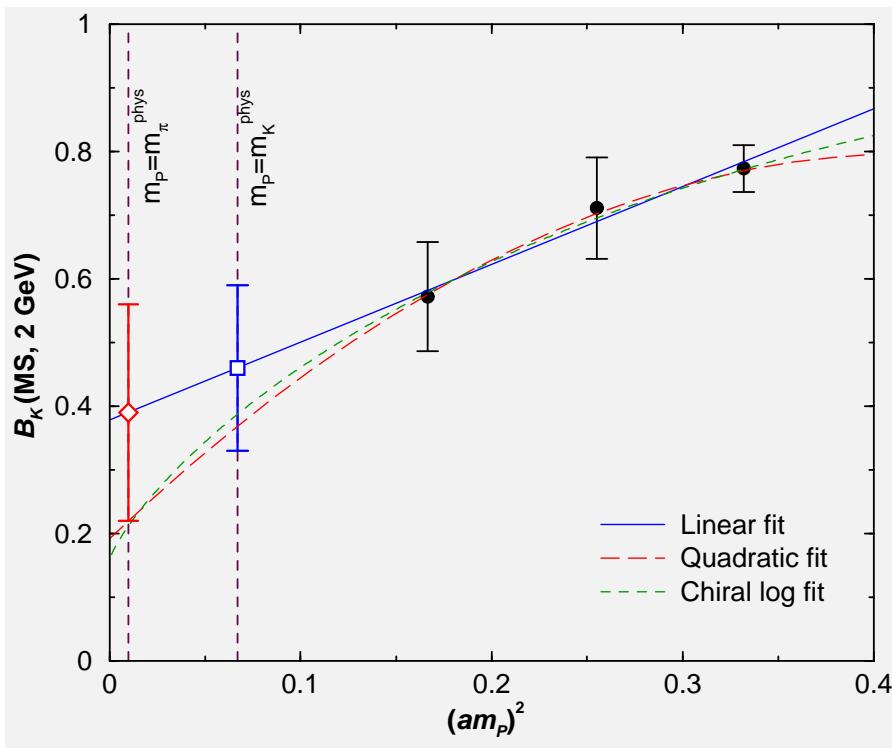
- Extrapolate **unitary** points to physical kaon



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- Extrapolate **unitary** points to physical kaon
- $B_K(\overline{\text{MS}}, 2 \text{ GeV}) = 0.48(13)$
- **Valence quarks realistic** (but degenerate)
- **Sea quarks light** but still around $m_s/2$

$N_f = 2 \Rightarrow$ lower value of B_K ?

Soni LAT95 review: unquenched numbers consistent with quenched, but always smaller.

Study	Action	$\frac{B_K(N_f=0)}{B_K(N_f=2)}$
Kilcup (1993)	Staggered	$\sim 1.09(7)$
Ishizuka <i>et al.</i> (1993)	Staggered	$\sim 1.03(5)$
Lee and Klomfass (1995)	Staggered	$\sim 1.06(10)$
Bernard and Soni (1995)	Wilson	$\sim 1.08(15)$

$N_f = 2 \Rightarrow$ **lower value of B_K ?**

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Remains valid in subsequent works, eg Kilcup *et al.* (1997) and Becirevic *et al.* (2000)

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- When sea quarks are finite-mass but heavy, B_K lower but still within errors of quenched
- When sea quarks are light, B_K distinctly lower than quenched

Conclusions

- Our number: $B_K(\overline{\text{MS}}, 2 \text{ GeV}) = 0.48 - 0.49(13)$
- close to: RBC prelim value $0.50(2)$ Izubuchi, [hep-lat/0310058](#)
free UT fit value $0.49(8)$ Ciuchini *et al.*, [hep-ph/0307195](#)
- There are concerns regarding the robustness of the numbers due to lattice artifacts and we *do not stress the value* obtained
- **Dynamical quark effects** seem to be **significant**: leading to **lower** value of B_K
- Earlier works could not see this only because their sea quarks were heavy and they could not extrapolate
- Details in [hep-lat/0406013](#)